CSE 125
Discrete Mathematics

Nazia Sultana Chowdhury,
Lecturer, Metropolitan University,
Sylhet.

nazia.nishat1971@gmail.com

Discrete Mathematics

- The study of mathematical structures that are fundamentally discrete rather than continuous.
- Countable or distinct and separable.
- Example: Combinations, graphs, and logical statements.

Why Discrete Mathematics

- Computer Network
- Computer algorithms
- Computer Program
- Programming Language

Why Discrete Mathematics

- Cryptography
- Automated Software Development
- Advanced Data Structure

Logic and Proofs

- Logic is the basis of all mathematical reasoning, and of all automated reasoning.
- Proof is an logical argument to validate a mathematical statement.

Practical applications of Logic

- The design of computing machines
- The specification of systems
- Artificial intelligence
- Computer programming
- Programming languages
- Other areas of computer science, as well as to many other fields of study.

Applications of Proofs

- To verify that computer programs produce the correct output for all possible input values
- To show that algorithms always produce the correct result
- Establishing the security of a system
- Creating Artificial intelligence

Propositional Logic

- Basic building blocks of logic—propositions.
- A proposition is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both.

Examples on Propositions

1. Washington, D.C., is the capital of the United States of America.

2. Toronto is the capital of Canada.

3. 1 + 1 = 2.

4. 2 + 2 = 3.

Propositions 1 and 3 are true, whereas 2 and 4 are false.

Examples on Propositions

1. What time is it?

2. Read this carefully.

3. $x + 1 = 2$.

4. $x + y = z$.

These are not Propositions

Proposition Convention

- Conventional letters used for propositional variables are p, q, r, s etc.
- The truth value of a proposition is true, denoted by T, if it is a true proposition
- The truth value of a proposition is false, denoted by F, if it is a false proposition.

- Let p be a proposition. The negation of p, denoted by¬p (also denoted by p), is the statement “It is not the case that p.”
- The proposition ¬p is read “not p.” The truth value of the negation of p, ¬p, is the opposite of the truth value of p.

Example

Find the negation of the proposition “She is happy” and express this in simple English.

Solution: The negation is “It is not the case that she is happy”. This negation can be more simply expressed as “She is not happy”.

Truth Table

- A truth table is a mathematical table used in logic—specifically in Boolean algebra, boolean functions, and propositional calculus.
- A truth table represents a table having all combinations of inputs and their corresponding result.

Truth table for the negation of a proposition p.

TABLE 1 The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

Connectives

- The logical operators that are used to form new propositions from two or more existing propositions.
- And , or , not etc. are such connectives.

Conjunction of proposition

Let p and q be propositions. The conjunction of p and q, denoted by $p \land q$, is the proposition "p and q."

The conjunction $p \land q$ is true when both p and q are true and is false otherwise

Example

Find the conjunction of the propositions p and q where p is the proposition “Dhaka is the capital of Bangladesh” and q is the proposition “Dhaka is the most populated city in Bangladesh”.

Solution: The conjunction of these propositions, $p \wedge q$, is the proposition “Dhaka is the capital of Bangladesh and the most populated city in Bangladesh”.

Truth Table

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

Let p and q be propositions. The disjunction of p and q, denoted by $p \vee q$, is the proposition "p or q." The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

Use of the connective or in a disjunction

- Inclusive or
- Exclusive or

Inclusive and Exclusive

- “Students who have taken calculus or computer science can take this class.”
- “Students who have taken calculus or computer science, but not both, can enroll in this class.

Truth Table

TABLE 3 The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive Or

Let p and q be propositions. The exclusive or of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise

Exclusive Or Truth Table

TABLE 4 The Truth Table for the Exclusive Or of Two Propositions.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional Statements

Let p and q be propositions. The conditional statement $p \rightarrow q$ is the proposition "if p, then q." The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise.

Conditional Statements

In the conditional statement $p \rightarrow q$, p is called the hypothesis (or antecedent or premise)
and q is called the conclusion (or consequence).

Expressing Conditional Statement

"if p, then q" "if p, q" "p is sufficient for q" "q if p" "q when p" "a necessary condition for p is q" "q unless ¬p"	"p implies q" "p only if q" "a sufficient condition for q is p" "q whenever p" "q is necessary for p" "q follows from p"

Examples of Conditional Statement

- “If I am elected, then I will lower taxes.”
- “If you get 100% on the final, then you will get an A.”

Truth Table for Condition Statement

TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Hypothesis and Conclusion Relation

- “If Juan has a smartphone, then 2 + 3 = 5”
- “If Juan has a smartphone, then 2 + 3 = 6”

CONVERSE, CONTRAPOSITIVE, AND INVERSE

Original Statement: $p \rightarrow q$

Converse: $q \rightarrow p$

Inverse: $\neg p \rightarrow \neg q$

Contrapositive: $\neg q \rightarrow \neg p$

Example

What are the contrapositive, the converse, and the inverse of the conditional statement

“The home team wins whenever it is raining?”

Solution: Because “q whenever p” is one of the ways to express the conditional statement $p \rightarrow q$, the original statement can be rewritten as

“If it is raining, then the home team wins.”

Consequently, the contrapositive of this conditional statement is “If the home team does not win, then it is not raining.”

The converse is “If the home team wins, then it is raining.”

The inverse is “If it is not raining, then the home team does not win.”

Only the contrapositive is equivalent to the original statement.

BICONDITIONALS

- Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition "p if and only if q." The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise.
- Biconditional statements are also called bi-implications.
- $p \leftrightarrow q$ has exactly the same truth value as $(p \rightarrow q) \land (q \rightarrow p)$

Expressing $p \leftrightarrow q$

"p is necessary and sufficient for q"

"if p then q, and conversely"

"p iff q."

Example

- Let p be the statement “You can take the flight,” and let q be the statement “You buy a ticket.”

  Then $p \leftrightarrow q$ is the statement “You can take the flight if and only if you buy a ticket.”

Truth Table of Biconditionals

TABLE 6 The Truth Table for the Biconditional $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Problem

- Construct the truth table of the compound proposition $(p \lor \neg q) \rightarrow (p \land q)$

Solution

TABLE 7 The Truth Table of $(p \lor \neg q) \rightarrow (p \land q)$.

p	q	$\neg q$	$p \lor \neg q$	$p \land q$	$(p \lor \neg q) \rightarrow (p \land q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Precedence of Logical Operators

- ¬p ∧ q is the conjunction of¬p and q, namely, (¬p) ∧ q,
  not the negation of the conjunction of p and q, namely¬(p ∧ q)
  p ∧ q ∨ r means (p ∧ q) ∨ r rather than p ∧ (q ∨ r)
- Conditional and biconditional operators → and ↔
  have lower precedence than the conjunction and disjunction operators,
  ∧ and ∨. p ∨ q → r is the same as (p ∨ q) → r.

Precedence of Logical Operators

TABLE 8
Precedence of Logical Operators.

Operator	*Precedence*
$\neg$	1
$\wedge$ $\vee$	2 3
$\rightarrow$ $\leftrightarrow$	4 5

Logic and Bit Operations

- Computers represent information using bits
- A bit is a symbol with two possible values, namely, 0 (zero) and 1 (one)
- A bit can be used to represent a truth value 1 represents T (true), 0 represents F (false). A variable is called a Boolean variable
- Computer bit operations correspond to the logical connectives

Truth Table for Bit Operators

TABLE 9 Table for the Bit Operators *OR*, *AND*, and *XOR*.

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

Problem

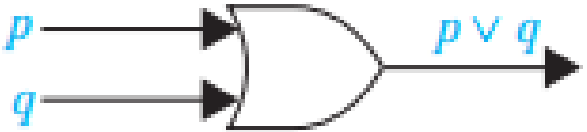
- Find the bitwise OR, bitwise AND, and bitwise XOR of the bit strings 01 1011 0110 and 11 0001 1101. (Here, and throughout this book, bit strings will be split into blocks of four bits to make them easier to read.)

Solution

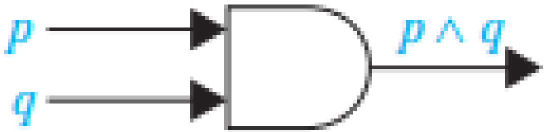
Solution: The bitwise *OR*, bitwise *AND*, and bitwise *XOR* of these strings are obtained by taking the *OR*, *AND*, and *XOR* of the corresponding bits, respectively. This gives us

01 1011 0110
11 0001 1101
―――――――――
11 1011 1111   bitwise *OR*
01 0001 0100   bitwise *AND*
10 1010 1011   bitwise *XOR*

Logical Gate



OR gate



AND gate

Propositional Equivalences

- A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a tautology.
- A compound proposition that is always false is called a contradiction.
- A compound proposition that is neither a tautology nor a contradiction is called a contingency

Logical Equivalences

- The compound propositions p and q are called logically equivalent if $p \leftrightarrow q$ is a tautology.
- The notation $p \equiv q$ denotes that p and q are logically equivalent

Thank You